Proof that $C_3 = 1.443 - (\partial C / \partial p_1 + \partial C / \partial p_2)/2$

(where C_3 is the 0 or 1 characters required for one of three events)

Given that:

p_i : probability of occurrence of event i

C_i : number of 0 or 1 characters required per event i

$C_i = -\log_2 p_i = -\ln p_i / \ln_2$

$\overline{C}_i$:average 0 or 1 characters required per event i

$\overline{C}_i = -p_i \log_2 p_i$

$\overline{C}_i = -p_i \ln p_i / \ln_2$

$\overline{C}$: average 0 or 1 characters generated per event for all three events i

$\overline{C} = \overline{C_1} + \overline{C_2} + \overline{C_3}$

$\overline{C} = -p_1 \log_2 p_1 - p_2 \log_2 p_2 - p_3 \log_2 p_3$

$\overline{C} = p_1 \ln p_1 / \ln 2 - p_2 \ln p_2 / \ln 2 - p_3 \ln p_3 / \ln 2$

$\overline{C} = \{p_1 \ln p_1 - p_2 \ln p_2 - p_3 \ln p_3\} / \ln 2$

Then

$$C_3 = 1.443 - (\partial C / \partial p_1 + \partial C / \partial p_2)/2$$

Proof:

(see next page)

$$\overline{C} = \{-p_1 \ln p_1 - (1-p_1-p_3)\ln(1-p_1-p_3) - (1-p_1-p_2)\ln(1-p_1-p_2)\}/\ln 2$$
$$\partial\overline{C}/\partial p_1 = \{[-\ln p_1 - 1] + [\ln(1-p_1-p_3)+1] + [(1-p_1-p_2)\ln(1-p_1-p_2)+1]\}/\ln 2$$
$$\partial\overline{C}/\partial p_1 = \{-\ln p_1 + \ln(1-p_1-p_3) + \ln(1-p_1-p_2) + 1\}/\ln 2$$
$$\partial\overline{C}/\partial p_1 = \{-\ln p_1 + \ln p_2 + \ln p_3 + 1\}/\ln 2$$

$$C = \{-(1-p_2-p_3)\ln(1-p_2-p_3) - (p_2)\ln(p_2) - (1-p_1-p_2)\ln(1-p_1-p_2)\}/\ln 2$$
$$\partial C/\partial p_2 = \{[\ln(1-p_2-p_3)+1] + [-\ln(p_2)-1] + [(\ln(1-p_1-p_2)+1]\}/\ln 2$$
$$\partial C/\partial p_2 = \{\ln(1-p_1-p_3) - \ln p_2 + \ln(1-p_1-p_2) + 1\}/\ln 2$$
$$\partial C/\partial p_2 = \{\ln p_1 - \ln p_2 + \ln p_3 + 1\}/\ln 2$$

$$\partial C/\partial p_1 + \partial I/\partial p_2 = \{2\ln p_3 + 2\}/\ln 2$$
$$\partial C/\partial p_1 + \partial I/\partial p_2 - \{2\}/\ln 2 = \{2\ln p_3\}/\ln 2$$
$$(\partial C/\partial p_1 + \partial I/\partial p_2)/2 - \{1\}/\ln 2 = \{\ln p_3\}/\ln 2$$
$$\{\ln p_3\}/\ln 2 = (\partial C/\partial p_1 + \partial I/\partial p_2)/2 - \{1\}/\ln 2$$
$$C_3 = -\{\ln p_3\}/\ln 2 = -(\partial C/\partial p_1 + \partial C/\partial p_2)/2 + \{1\}/\ln 2$$
$$C_3 = -\{\ln p_3\}/\ln 2 = \{1\}/\ln 2 - (\partial C/\partial p_1 + \partial C/\partial p_2)/2$$
$$C_3 = -\{\ln p_3\}/\ln 2 = 1.443 - (\partial C/\partial p_1 + \partial C/\partial p_2)/2$$

Note: not only does the fourth to last equation above,

$$\{\ln p_3\}/\ln 2 = (\partial C/\partial p_1 + \partial I/\partial p_2)/2 - \{1\}/\ln 2,$$

express the log to the base 2 of any probability between 0 and 1, but also for any value y > 1, by setting p_3 to 1/y and p_1 and p_2 to arbitrary probabilities such that $1 = p_1 + p_2 + p_3$ and then using these values in the negative of the equation:

$$-[\{\ln 1/y\}/\ln 2 = (\partial C/\partial p_1 + \partial I/\partial p_2)/2 - \{1\}/\ln 2].$$